

Interacting Convection Zones

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Abstract. We present results from simulations to examine how the separation between two convectively unstable layers affects their interaction. We show that two convectively unstable layers remain connected via the overshooting plumes even when they are separated by several pressure scale heights.

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INTRODUCTION

Some main-sequence stars, such as A-type stars, have a complex internal structure in which there are believed to exist multiple convection zones (Toomre *et al.*, 1976; Kupka, 2005). Such a complicated internal structure arises because of the change in the chemical composition as the distance from the centre of the star is increased. The outermost convection zone, located just below the photophere is believed to be caused by the partial ionization of hydrogen and the single ionization of helium. The lower convection zone is believed to result from the second ionization of helium and is separated from the upper convection zone by a relatively narrow convectively stable region (Latour, Toomre & Zahn, 1976; Kupka, 2005).

The presence of two convectively unstable zones in such stars raises a number of interesting questions. However, in this short article we choose to focus on the principal question concerning the degree of interaction between the convectively unstable regions. Such an issue is motivated by the fact that in stars, without the presence of rigid boundaries, ascending and descending convectively driven motions overshoot the bottom of the layer that is convectively unstable. When there are multiple convection zones, in close proximity, in stars this overshooting behaviour leads to enhanced transport and communication between the unstable layers. Further, due to the close proximity of the unstable layers, fascinating dynamics may occur. For example, overshooting plumes from each unstable layer could overshoot so far that they continue through the stable region and pierce the other convection zone, which would lead to transportation of ‘contaminants’ directly from one convectively unstable region to the other (Tobias *et al.*, 1998, 2001). In this article we will present some results from simulations to explore how increasing the separation affects the connection between the layers.

It has been conjectured, from analytic theory, that two pressure scale heights should be sufficient to ‘disconnect’ the two convectively unstable regions (Latour, Toomre & Zahn, 1976; Toomre *et al.*, 1976). Here, we will briefly show that this is an underestimate and that to ‘disconnect’ the two convectively unstable layers many more scale heights of separation is required. A more detailed exploration

of this issue can be found in Silvers & Proctor (2007).

MODEL

Here we examine the evolution of a compressible fluid in a Cartesian layer. The governing equations are (Matthews, Proctor, & Weiss, 1995):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{u} = 0, \quad (1)$$

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla P + \theta(m+1)\rho \hat{\mathbf{z}} + \sigma \kappa \nabla \cdot \rho \boldsymbol{\tau}, \quad (2)$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = -(\gamma-1)T \nabla \cdot \mathbf{u} + \frac{\kappa(\gamma-1)\sigma \tau^2}{2} + \frac{\gamma \kappa}{\rho} \nabla^2 T, \quad (3)$$

where z is taken downward, θ is the dimensionless temperature difference across the layer, R_* is the gas constant, m is the polytropic index, $\kappa = K/d\rho_0 c_P \sqrt{(R_* T_0)}$ is the dimensionless thermal diffusivity, γ is the ratio of specific heats, $\boldsymbol{\tau}$ is the stress tensor, $P = \rho T$ and σ is the Prandtl number. These equations above are solved using a parallel hybrid finite-difference/pseudo-spectral code; the most comprehensive description of the code can be found in (Matthews, Proctor, & Weiss, 1995).

To obtain a suitable basic state for this problem we allow the thermal profile to be non-linear and we take

$$K = \frac{K_1}{2} \left[1 + \frac{K_2 + K_3}{K_1} + \frac{K_3}{K_1} \tanh \left(\frac{z - \mathcal{D} + 1}{\Delta} \right) - \left(\frac{K_2}{K_1} \tanh \left(\frac{z - \mathcal{D} + 1}{\Delta} \right) + 1 \right) \tanh \left(\frac{z - 1}{\Delta} \right) \right] \quad (4)$$

where Δ is the characteristic size of the transition region between each of the layers. In this work the characteristic sizes of the transition regions are taken to be the same for simplicity. The static density and temperature profiles are found by solving the equations of hydrostatic balance. To this static state, throughout the domain, random perturbations are introduced, with amplitudes which lie within the interval $[-0.05, 0.05]$.

The aspect ratio for the computational domain in this study is 8:8: \mathcal{D} ¹, where \mathcal{D} is the total depth of the box, and the domain is assumed to be periodic in x and y . The conditions on the upper and lower boundaries are:

$$T = 1, \quad u_z = 0, \quad \frac{\partial u_x}{\partial z} = 0 \quad \text{at } z = 0. \quad (5)$$

$$\frac{\partial T}{\partial z} = \theta, \quad u_z = 0, \quad \frac{\partial u_x}{\partial z} = 0. \quad \text{at } z = \mathcal{D}. \quad (6)$$

¹ Note that the more detailed exploration of this topic in (Silvers & Proctor, 2007) is in a, smaller, 4:4: \mathcal{D} domain.

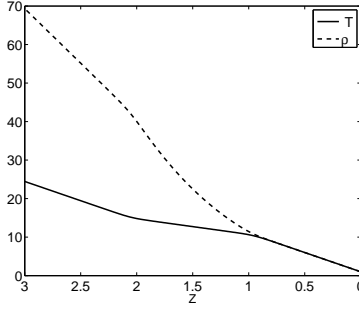


FIGURE 1. The initial temperature and density profiles when $\mathcal{D} = 3.0$.

The system we study has a large number of dimensionless parameters, making it impractical to conduct a complete survey. Thus a number are held fixed: $\sigma = 1.0$, $m_1 = m_3 = 1.0$, $\theta = 10.0$, $\gamma = 5/3$ and $R_a = 1.7 \times 10^5$. Note that in this paper we use a subscript 1 on quantities relating to the upper convection zone. Similarly, subscript 2 will be used to denote quantities for the convectively stable layer and 3 to denote quantities in the lower convective zone.

The stiffness parameter, S , provides a useful measure of the relative conductivities in this problem (for more detailed discussion see Tobias *et al.* (1998)). S_2 and S_3 are related to the various polytropic indexes that appear in the problem via the definition that $S_2 = (m_2 - m_{ad})/(m_{ad} - m_1)$ and $S_3 = (m_3 - m_{ad})(m_{ad} - m_1)$.

THE EFFECT OF VARYING THE THICKNESS OF THE STABLE LAYER

We begin by considering the case where all three zones have equal depth and so $\mathcal{D} = 3.0$. The static state profiles for this fiducial model are shown in Figure 1. Note that for this fiducial case there are 1.60 pressure scale heights across the convectively stable mid-layer, which is rather less than the two pressure scale heights estimate suggested by the earlier analytic theory as necessary for true separation.

The state shown in Figure 1 is perturbed and allowed to evolve to a statistically steady state as shown in Figure 2. As one might expect the convection is noticeably different in the two convection zones. While Figure 2 provides a useful picture of the convective state it is difficult to gauge the motions within the convectively stable region from such a figure. To obtain a clearer picture of the potential interactions between the two convectively unstable regions it is helpful to note that, if the two layers are to be considered as ‘independent’ from each other, then there needs to be a region between the layers where the velocity becomes very small. Therefore, to facilitate a clearer picture of the degree of interaction of the layers we calculate the variation in z of the horizontal averages of the modulus of the z component of momentum.

Figure 3(a) shows that while there is clearly more vertical motion in the convectively unstable regions, as one might anticipate, there is still a non-negligible vertical component of momentum in the middle of the box. Thus the two convection zones are con-

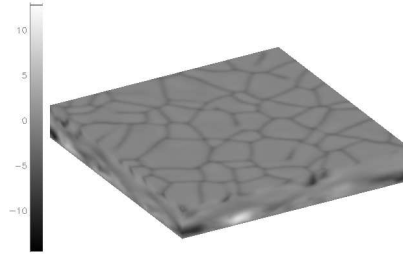


FIGURE 2. Plot at $t = 15.02$ Sides of the box show the vertical momentum flux and the top of the box shows the vertical momentum flux near the top of the box for the case where $\mathcal{D} = 3.0$ and $S_2 = 5.0$.

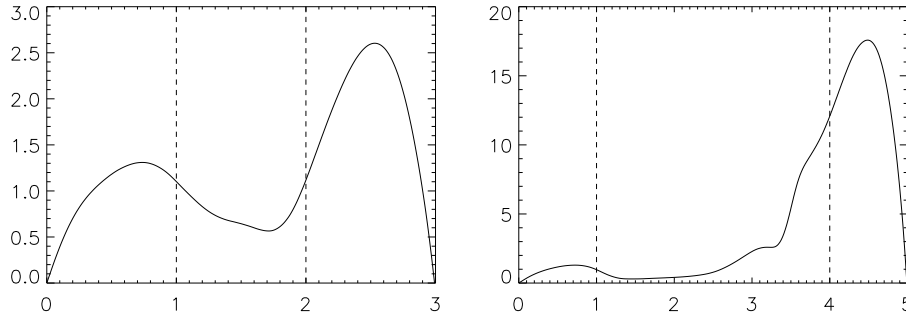


FIGURE 3. Left: Plot at $t = 15.32$. The variation of the vertical component of velocity as a function of height at the centre of the box for the case where $\mathcal{D} = 3.0$ and $S_2 = 5.0$. Right: Plot at $t = 15.09$. The variation of the vertical component of velocity as a function of height at the centre of the box for the case where $\mathcal{D} = 5.0$ and $S_2 = 5.0$.

nected in this case and so there is a conduit for mixing between the two convection zones for this level of separation. Any reduction in the width of the convectively stable region will decrease the number of pressure scale heights of variation across the layer and increase the level of interaction between the two unstable zones. Such vigorous motion in the convectively stable layer was discussed in the context of the downward directed hexagon case of Latour *et al.* and is further confirmed by a plot of the vertical component of velocity as shown in 3(a).

The most obvious way to limit the interaction of the unstable regions is to increase the layer depth, \mathcal{D} , and so increase the width of the stable region if the convectively unstable regions have the same height. We have gradually increased the layer depth up to 5.0. This corresponds to a pressure scale height variation of 3.77 across the stable layer, which is above the estimate suggested by the earlier analytic theory as the transition between connected and unconnected convection layers.

As for the case when $\mathcal{D} = 3.0$, we consider the variation of the planar average vertical component of momentum throughout the domain in the established statistically steady state, for each of the increased box heights. Figures 3(b) show this quantity for each case when $\mathcal{D} = 5.0$. This figure clearly illustrates that, while there is a significant reduction

in vertical motion in the convectively stable region, there is a non negligible vertical component of momentum throughout the box even for a separation of 3.77 pressure scale heights.

CONCLUSIONS

In this article we have shown that there is no point at which the vertical component of momentum becomes very small even when the number of pressure scale heights of separation between the two convectively unstable layers is as large as 3.77. Thus there is a route for mixing of passive and dynamic quantities between the two convectively unstable regions in all the cases we considered. Only for a substantial increase in the box height will the vertical component of momentum fall to a very small value at one plane in the box. To find the minimum box height for this to occur would be extremely computationally expensive and also uninteresting physically. In A-type main sequence stars the separation between the unstable layers does not extend past a couple of scale heights. Therefore, on the basis of the present work we can conclude that in A-type stars there is a clear connection between the convectively unstable zones that lie immediately below the stellar photosphere.

One of the major objectives of this work is to provide a solid hydrodynamical basis on which more complex models can be constructed to understand fully the dynamics that occur below the surface of A-type stars. With such a simple model we are not yet in a position to address important secondary questions such as the influence of the observed chemical anomalies (see discussions in Michaud (1970) or Vauclair & Vauclair (1982) for more details). However, we believe our model provides a platform on which we can build, so as to address the effects of rotation, magnetic fields and other issues related to the dynamics in these stars and will be the subject of future work.

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